

# Mathematics

The mathematical basis behind ARTIS is publicly accessible.

The book of Hoyland and Rausand, *System Reliability Theory: Models and Statistical Methods*, Wiley, Hoboken, 1994, provides the Reliability Centered Maintenance background.

The Master's Thesis of Lukasz Bednara, *Methods for Approximating the Availability Functions*, Master's thesis, TU Delft, 2008, completes the mathematical analysis.

## Model concepts

An availability diagram is a graphical representation of the equipment items of the system, showing their impact on the system availability. The model diagram shows how the system depends on each item for its proper functioning. The diagram is not a process flow scheme, it does not show the flow of product and indeed items from different systems may be shown connected. Items may not necessarily be shown in the order in which they occur in the process.

Items are in series when their proper functioning is vital for plant performance; if one item fails, the entire process stops. They are placed in parallel if upon failure of one item, the other parallel item(s) can (partly) take over and the process continues, possibly at a lower capacity. Parallelism is not restricted to single items, e.g. the compression trains in LNG liquefaction are placed in parallel, each containing several compressors, heat exchangers and knock out drums.

Each item has a maximum operating capacity and inherits its downtime data from its unit, Each unit represent a set of comparable items that share the following data:

- by-pass capacity upon failure (e.g. a manual bypass on a valve)
- failure data: Mean Time To Failure (MTTF) and Mean Time To Repair (MTTR)
- preventive maintenance or inspection strategy

For a given period, time is split into 4 different categories: operation time, standby time, planned downtime and unplanned downtime. For a single item with constant capacity, the simple definitions of its availability and reliability are:

- Availability = (operation time + standby time) / total time, a unit is available if it is either in operation or on standby.
- Reliability = (operation time + standby time) / (total time - planned downtime), the reliability of a unit is defined as the fraction of time that the unit is working when required, that is outside the planned downtime periods.

The only difference is that availability includes the planned downtime and the reliability does not. During periods with no planned downtime, the two are equal. These definitions are consistent with the ISO 14224 standard, which describes availability as: "the ability of an item to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval, assuming that the required external resources are provided".

With parallel items and bypass capacities, a system may have many possible capacity levels. Therefore, the definitions of availability and reliability are generalised, using the Available Capacity: that is a table of all possible capacity levels and the fraction of time spent at each level. From the available capacity, we can derive how much capacity can be expected from the system in one year. This is called Expected Capacity:

- Expected Capacity =  $\sum \text{capacity [i]} * (\text{fraction of time spent at capacity [i]})$ .

The general definitions of Availability and Reliability during a given period with capacity fluctuations are:

- Availability = Expected Capacity / Maximum Capacity.
- Reliability = Expected Capacity / (Maximum capacity - Planned Loss).

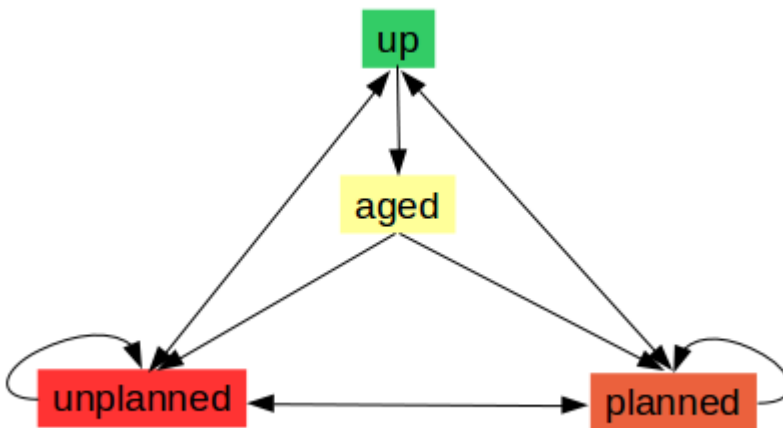
In the general definitions of Availability and Reliability, the Maximum Capacity stands for the maximum operating capacity, not the nameplate or design capacity. In case of capacity fluctuations, such as production profiles and seasonal swings, the maximum operating capacity is the time-weighted average. The Planned Loss is the loss that is due to planned downtime.

## State transitions

After the unit starts up, each downtime mode cycles through four possible states and each state has its own availability, as a function of time. Whenever the mode is known to go up or down and at the time Now, a state transition occurs.

The states are:

1. up
2. unplanned down
3. planned down
4. aged, that is for modes with Weibull lifetime that have been up for some time, at the time Now



The timing of the state transitions is:

1. at the time Now
2. at the start of a planned event, this time must be known
3. at the start of an unplanned event, this time must also be known
4. at the end of an event, if that time is known

Before a unit starts up, all its modes are assumed to be down.

## Fault Tree Analysis

ARTIS applies all the availability modelling definitions to fault tree analysis, by replacing 'capacity' and 'production', on a continuous scale, by 'mission', on a binary scale. This gives rise to a set of definitions of well-established terms like 'mission reliability' and 'mission criticality' that is consistent with common practice in fault tree analyses.

A production availability model can be interpreted as a success tree by replacing the capacities (non-negative real numbers) by states (true or false), taking the serial trains as AND gates and taking the parallel trains as OR gates.

A success tree is logically equivalent to a fault tree by de Morgan's theorem. The top event of the fault tree represents the set of system states with capacity 0 in the availability model. The cut sets are the minimum sets of downtime events of all these system states.

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